A. L. ZUYEV [1]

Mathematical Control Theory in Real World Applications

[1] *Institute of Applied Mathematics and Mechanics, National Academy of Sciences of Ukraine, Donetsk, Ukraine*
E-mail: alz@iamm.ac.donetsk.ua, al_zv@mail.ru

This lecture is focused on recent applications of mathematical methods for solving a challenging social problem related to the optimization of traffic flows. The importance of such innovative approach to transport problems is clearly illustrated by the economic impact of the timetable optimization for the Dutch Railways (Nederlandse Spoorwegen). In 2007, the Dutch Railways introduced the new timetable based on mathematical optimisation models [1]. This innovation resulted in an additional annual profit of 40 million Euros. Another illustrious example is given by the Active Traffic Management (ATM) technology of increasing peak capacity on highways. On the M42 motorway in the UK, the ATM allows to decrease journey times by 26% in the northbound direction [2].

In this lecture, macroscopic models of the traffic flow are considered [3]. For simplicity, we assume that the vector of densities of cars $\rho(x,t) \in \mathbb{R}^n$ on n lanes is governed by a generalized model of the Lighthill–Whitham–Richards type as follows:

$$\frac{\partial}{\partial t}\rho(x,t) + \frac{\partial}{\partial x}Q(\rho(x,t)) = \Psi(x)u, \quad x \in (0,l),\ t \geq 0, \tag{1}$$

where l is the length of a segment of the road, x is the spatial coordinate, t is the time, and $u \in \mathbb{R}^m$ is the control. The flux $Q : \mathbb{R}^n \to \mathbb{R}^n$ is related to the fundamental traffic diagram, and the $n \times m$-matrix $\Psi(x)$ describes junctions on the road. We address the optimal control problem for system (1) with periodic boundary conditions. A possibility of reducing the traffic jam by controlling the flow at junctions is discussed.

[1] A. Schrijver, Flows in railway optimization, *Nieuw Archief voor Wiskunde* **9**, No. 2 (2008), P. 126–131.

[2] *ATM Monitoring and Evaluation, 4-Lane Variable Mandatory Speed Limits* (12 Month Report), Highways Agency, Bristol, UK, 2008.

[3] R. Haberman, *Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow*, SIAM, Philadelphia, 1998.