

On elliptic systems on the extended Sobolev scale

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The talk is devoted to applications of some spaces of generalized smoothness to general elliptic systems of partial differential equations. These spaces form the extended Sobolev scale $\{H^\varphi(\Gamma) : \varphi \in \text{RO}\}$ over a closed (and compact) C^∞ -manifold Γ . Here, RO is the class of all Borel measurable functions $\varphi : [1, \infty) \rightarrow (0, \infty)$ such that $c^{-1} \leq \varphi(\lambda t)/\varphi(t) \leq c$ for every $t \geq 1$ and $\lambda \in [1, a]$, the constants $a > 1$ and $c \geq 1$ not depending on t and λ but can depend on φ . Such functions were introduced by V. Avakumović.

The Hilbert space $H^\varphi(\Gamma)$ consists of all distributions on Γ that belong in local coordinates to the isotropic Hörmander space $H^\varphi(\mathbb{R}^n)$, where $n := \dim \Gamma$. Here, $H^\varphi(\mathbb{R}^n)$ consists of all tempered distributions $w \in S'(\mathbb{R}^n)$ such that $\varphi(\langle \xi \rangle) (Fw)(\xi) \in L_2(\mathbb{R}^n, d\xi)$. As usual, Fw is the Fourier transform of w , and $\langle \xi \rangle := (1 + |\xi|^2)^{1/2}$.

The extended Sobolev scale consists of all Hilbert spaces that are interpolation spaces for couples of inner product Sobolev spaces.

For Douglis–Nirenberg elliptic systems, the following results are obtained [1]:

- 1) A theorem on the Fredholm property of the matrix elliptic operators on the extended Sobolev scale.
- 2) New a priori estimates for solutions to the elliptic systems.
- 3) A theorem on the local regularity of the solutions.
- 4) New sufficient conditions under which the generalized derivatives (of a prescribed order) of the solutions are continuous.

Other classes of elliptic systems are investigated on the extended Sobolev scale as well [2-4].

- [1] T. N. Zinchenko, *Reports of NAS of Ukraine*, (2013), no. 3, p. 14–20.
- [2] T. N. Zinchenko, A. A. Murach, *Ukrainian Math. J.* **64** (2013), no. 11, p. 1672–1687.
- [3] T. N. Zinchenko, A. A. Murach, *J. Math. Sci.* **196** (2014), no. 5, p. 721–732.
- [4] A. A. Murach, T. N. Zinchenko, *Methods Funct. Anal. Topology* **19** (2013), no. 1, p. 29–39.