

S. S. KOVALENKO <sup>1</sup>, O. M. PATSIUK <sup>2</sup>

## Symmetry reduction and exact solutions of the non-linear Black–Scholes equation

<sup>1</sup> *Institute of Mathematics, Kyiv, Ukraine*  
*E-mail: kovalenko@imath.kiev.ua*

<sup>2</sup> *Institute of Mathematics, Kyiv, Ukraine*  
*E-mail: patsyuck@yahoo.com*

We investigate the non-linear Black–Scholes equation (see [1],[2]):

$$u_t + ax^2u_{xx} + bx^3u_{xx}^2 + c(xu_x - u) = 0, \quad a, b > 0, \quad c \geq 0,$$

and show that one can be reduced to the equation

$$u_t + (u_{xx} + u_x)^2 = 0$$

by an appropriate point transformation of variables.

For the last equation, we study the group-theoretic properties, namely, we find the maximal algebra of invariance of its in Lie sense, carry out the symmetry reduction and seek for a number of exact group-invariant solutions of the equation.

[1] M. Avellaneda, A. Páras *Appl. Math. Finance* **1**, (1994), p. 165.

[2] U. Cetin, R. Jarrow, P. Protter *Finance Stoch.* **8**, (2004), p. 311.